HOMEWORK SOLUTIONS 6
Chapter 7
Frequency Response

Problems
7.1,7.3,7.4,7.5,
7.6,7.7,7.10,7.14,
7.15,7.19,7.26

Electronics II
ECGR 3132
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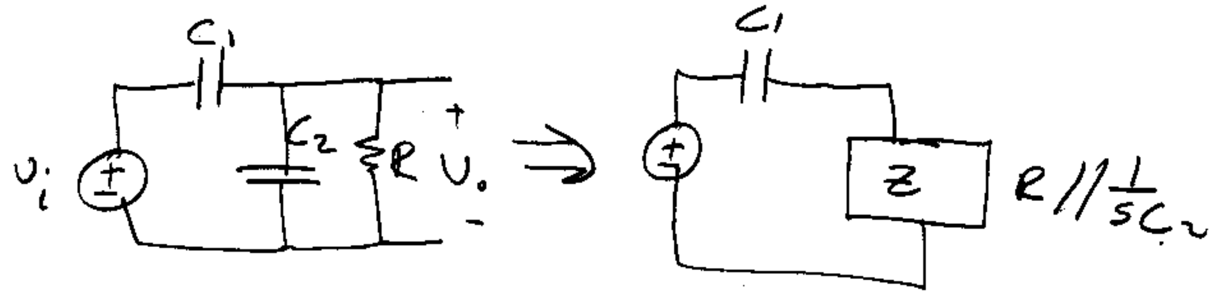
7.1



$$\frac{\frac{1}{sC_2} R}{R + \frac{1}{sC_2}} = \frac{\cancel{\frac{1}{sC_2}} R}{\cancel{\frac{1}{sC_2}}(1 + sRC_2)} = \frac{R}{1 + sRC_2}$$

Once we have the impedance of the parallel combination R and C_2 we can use voltage division to find the transfer function.

$$\frac{\frac{R}{1+sRC_2}}{\frac{R}{1+sRC_2} + \frac{1}{sC_1}} = \frac{\frac{R}{\cancel{1+sRC_2}}}{\frac{1}{\cancel{1+sRC_2}}\left(R + (1+sRC_2)\frac{1}{sC_1}\right)} = \frac{V_{out}}{V_{in}}$$

$$\frac{R}{R + \frac{1}{sC_1} + sRC_2\left(\frac{1}{sC_1}\right)} = \frac{R}{\frac{1}{sC_1}(sRC_1 + 1 + sRC_2)}$$

$$\frac{sRC_1}{1 + sR(C_1 + C_2)} = \frac{V_{out}}{V_{in}}$$

To find the zeros set numerator equal to zero, and solve for s

$sRC_1 = 0 \qquad s = 0$

so zero at 0

To find the poles set denominator equal to zero, and solve for s

$1 + sR(C_1 + C_2) = 0$

$$s = \frac{-1}{R(C_1 + C_2)} = \frac{-1}{100k(0.5\mu f + 0.5\mu f)} = -10\ \text{Rad/Sec}$$

To plot the Mag. and phase we can rewrite the equations

$$\frac{sRC_1}{1 + sR(C_1 + C_2)}$$

$$w_z = \frac{1}{RC_1} = 20\ \text{Rad/Sec}$$

$$w_p = \frac{1}{R(C_1 + C_2)} = 10\ \text{Rad/Sec}$$

because $s = jw$ we can write the equation

$$\frac{j\frac{w}{20}}{1 + j\frac{w}{10}}$$

Since we know that the Bode plot starts increasing $20\,dB/dec$ because of the zero at zero we can plug in a frequency higher than $10\,rad/sec$ and get the max mag.

$100\,rad/sec$ was chosen

$$\frac{j\frac{100}{20}}{1 + j\frac{100}{10}}$$ then $Mag = \sqrt{R^2 + I^2}$

real

imaginary

$$Phase = \angle \tan^{-1}\left(\frac{I}{R}\right)$$

$$Mag_{max} = \frac{\sqrt{5^2}}{\sqrt{1^2 + (10)^2}} = 0.5$$

* These are fundamental equations for electrical engineers and you should know them well

$$20 \log 0.5 = -6\,dB$$

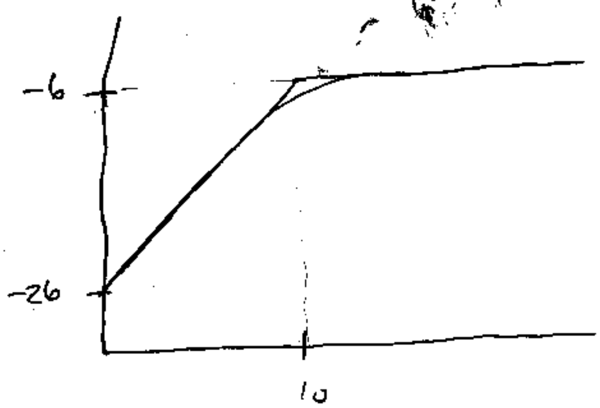
$$Mag_{max} = -6\,dB$$

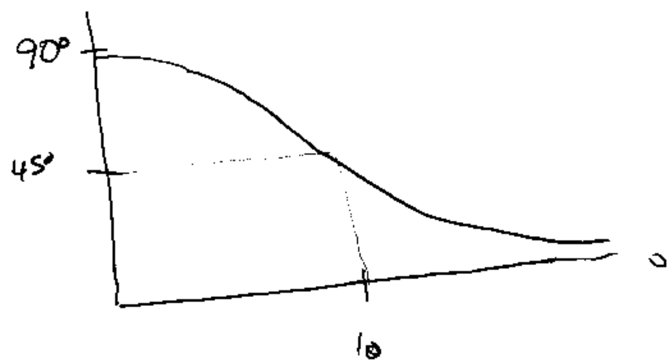
to find the phase just use the 3dB cutoff frequency

$$\frac{j\frac{10}{20}}{1+j\frac{10}{10}} = \frac{\angle \tan^{-1}\frac{\frac{1}{2}}{0}}{\angle \tan^{-1}\left(\frac{1}{1}\right)} = \frac{\angle 90}{\angle 45} = \angle -45^\circ$$

$\swarrow \infty$

So our Bode plot will look like this





7.3) There are three possible situations.

① Both are Low pass LP
② Both are High pass HP
③ one is LP and the other is LP

① Both are Low pass, Because they are equal they will have the same transfer function.

$$T_1(s) = T_2(s) = \frac{1}{1 + j\frac{\omega}{\omega_p}} = \frac{1}{1 + j\frac{\omega}{100}}$$

To find the total gain we have to multiply $T_1(s)$ and $T_2(s)$ together.

$$T(s) = T_1(s)T_2(s) = \left(\frac{1}{1 + j\frac{\omega}{100}}\right)^2$$

Now take the mag and plug in frequency

$$|T(s)| = \left[\frac{1}{\sqrt{1 + (\frac{10}{100})^2}}\right]^2 = 0.99 \text{ @ } 10 \text{ rads/sec}$$

$$T(s) = \left[\frac{1}{\sqrt{1 + (\frac{100}{100})^2}}\right]^2 = 0.5 \text{ @ } 100 \text{ rads/sec}$$

$$T(s) = \left|\frac{1}{\sqrt{1 + (\frac{1000}{10})^2}}\right| = 0.01 \text{ @ } 1000 \text{ rads/sec}$$

② Both are HP

$$T_1(s) = T_2(s) = \frac{s}{s+\omega} = \frac{1}{1+\frac{\omega}{s}} = \frac{1}{1+\frac{100}{s}}$$

$$T(s) = \left(\frac{1}{1+\frac{100}{j\omega}}\right)^2 = \left(\frac{1}{1-j\frac{100}{\omega}}\right)^2$$

$$|T(s)| = \left[\frac{1}{\sqrt{1^2+\left(\frac{100}{\omega}\right)^2}}\right]^2 = 0.01 \text{ @ } 10 \text{ rads/sec}$$

$$T(s) = \left[\frac{1}{\sqrt{1^2+\left(\frac{100}{100}\right)^2}}\right]^2 = 0.5 \text{ @ } 100 \text{ rads/sec}$$

$$T(s) = \left[\frac{1}{\sqrt{1^2+\left(\frac{100}{1000}\right)^2}}\right]^2 = 0.99 \text{ @ } 1000 \text{ rads/sec}$$

③ one LP and other HP

$$T_1(s) = \frac{1}{1+\frac{s}{100}} \qquad T_2(s) = \frac{1}{1+\frac{100}{s}}$$

$$T(s) = \left(\frac{1}{1+\frac{s}{100}}\right)\left(\frac{1}{1+\frac{100}{s}}\right) = \left[\frac{1}{1+j\frac{\omega}{\omega_1}}\right]\left[\frac{1}{1-j\frac{\omega_2}{\omega}}\right]$$

$$|T(s)| = \left[\frac{1}{\sqrt{1+\left(\frac{10}{100}\right)^2}}\right]\left[\frac{1}{\sqrt{1+\left(\frac{100}{10}\right)^2}}\right] = 0.1 \text{ @ } 10 \text{ rad/sec}$$

$$T(s) = \left[\frac{1}{\sqrt{1+\left(\frac{100}{100}\right)^2}}\right]\left[\frac{1}{\sqrt{1+\left(\frac{100}{100}\right)^2}}\right] = 0.5 \text{ @ } 100 \text{ rad/sec}$$

$$T(s) = \left[\frac{1}{\sqrt{1+\left(\frac{1000}{100}\right)^2}}\right]\left[\frac{1}{\sqrt{1+\left(\frac{100}{1000}\right)^2}}\right] = 0.1 \text{ @ } 1000 \text{ rad/sec}$$

7.4

$$T(s) = \frac{a_1 s}{s + \omega_0}$$

High Frequency gain = a_1 = 10 V/V

$\therefore a_1 = 10$

At $s = j2\pi(10)$, $|T| = 1$

Thus $$\left.\frac{a_1}{\sqrt{1+\left(\frac{\omega_0}{\omega}\right)^2}}\right|_{\omega = 2\pi(10)} = 1$$

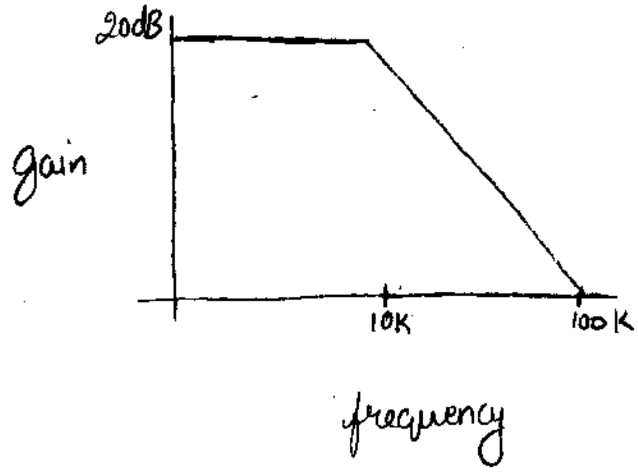
$$\Rightarrow \sqrt{1+\left(\frac{\omega_0}{\omega}\right)^2} = a_1 = 10$$

$$\omega_0 = 2\pi(10)(\sqrt{99})$$

$$= 625 \text{ rad/s}$$

$$(f_0 \simeq 100\text{Hz})$$

7.5



Since we decrease by 20 dB at a pole and we know that 100K is 0dB and low frequeny is 20 dB This leads to 10 KHz corner frequency.

$$T(s) = \frac{10}{1 + \frac{s}{2\pi(10^4)}}$$

$$|T(j\omega)| = \frac{10}{\sqrt{1+(\omega/2\pi(10^4))^2}}$$

$$= \frac{10}{\sqrt{1+(f/10^4)^2}}$$

$$20 \log |T| = 20 - 10 \log\left(1 + \frac{f^2}{10^8}\right)$$

Since given gain $= 19\,dB$

$$19 = 20 - 10\log\left(1 + \frac{f^2}{10^8}\right)$$

$$1 + \frac{f^2}{10^8} = 10^{0.1}$$

$$f = 10^4 \sqrt{10^{0.1} - 1}$$

$$= 5088.5 \text{ Hz}$$

$$\phi = -\tan^{-1}(f/10^4)$$

$$-6^\circ = -\tan^{-1}(f/10^4)$$

$$f = 1051 \text{ Hz}$$

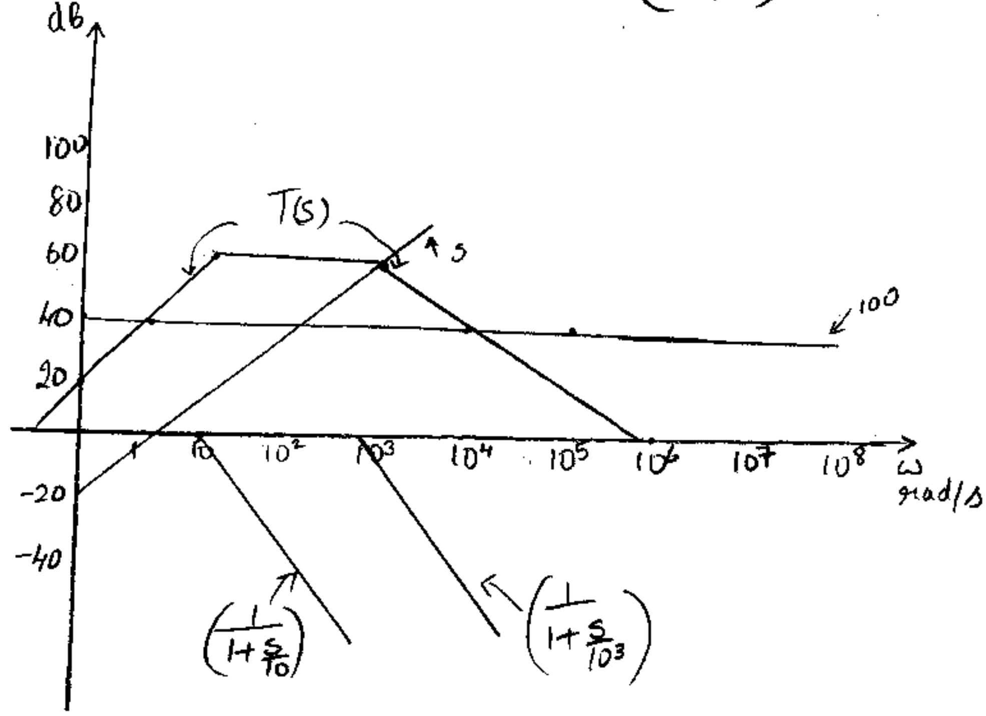
7.6

Other poles must be at $-7-j10$ (Complex Conjugate)

Other zeros must be at $-1+j20$ (Complex Conjugate)

7.7 Convenient form for constructing Bode Plots

$$T(s) = \frac{100s}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{10^3}\right)}$$



From Bode Plot:

ω	1	10	10^2	10^3	10^4	10^5
$\lvert T \rvert$	40	60	60	60	40	20

From Transfer function :-

Gain at 10 rad/s

$$|T(j10)| = \frac{100 \times 10}{\sqrt{1 \times 1}\ \sqrt{1+(0.01)^2}} \simeq \frac{1000}{\sqrt{2}}$$

$\Rightarrow$ 57 dB

Gain at 10^3 rad/s

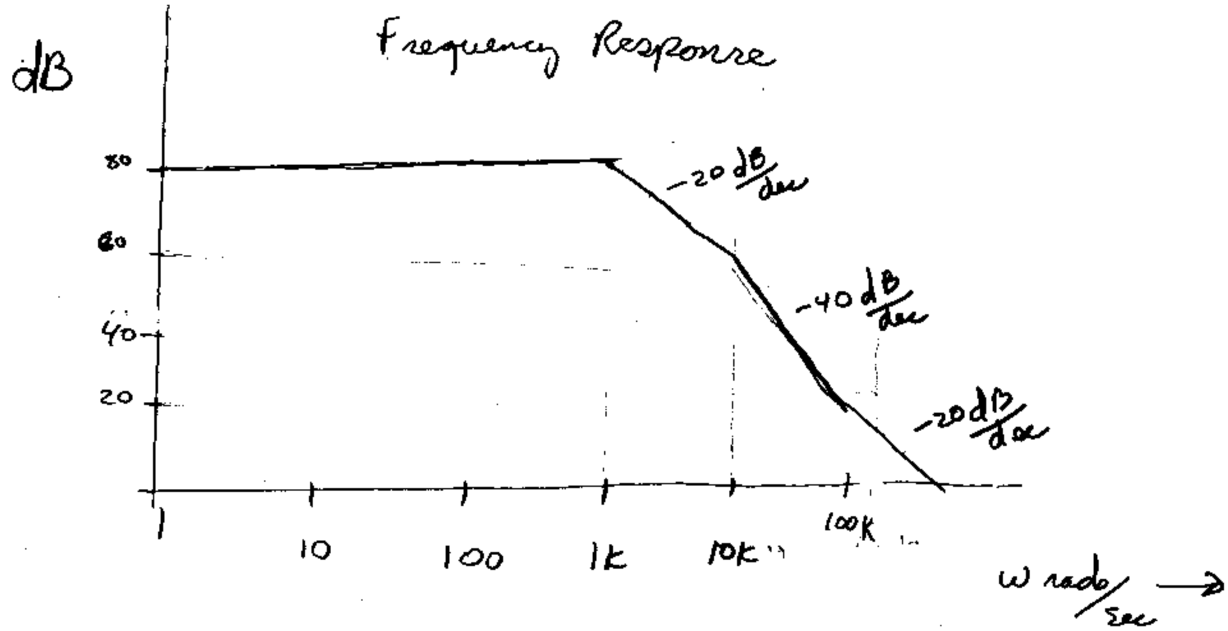
$$|T(j10^3)| = \frac{100 \times 10^3}{\sqrt{1 \times 10^4}\ \sqrt{1 \times 1}} \simeq \frac{1000}{\sqrt{2}}$$

$\Rightarrow$ 57 dB

7.10) $T(s) = \dfrac{10^4(1 + s/10^5)}{(1 + \frac{s}{10^3})(1 + \frac{s}{10^4})}$

Since max gain will be 10^4 this is where we start

Frequency Response



$A_v = 10^4 = 80dB$

The first and dominate pole is at 1k

$1 + \frac{s}{10^3} \leftarrow \omega_{p_1}$

Next pole is at 10K

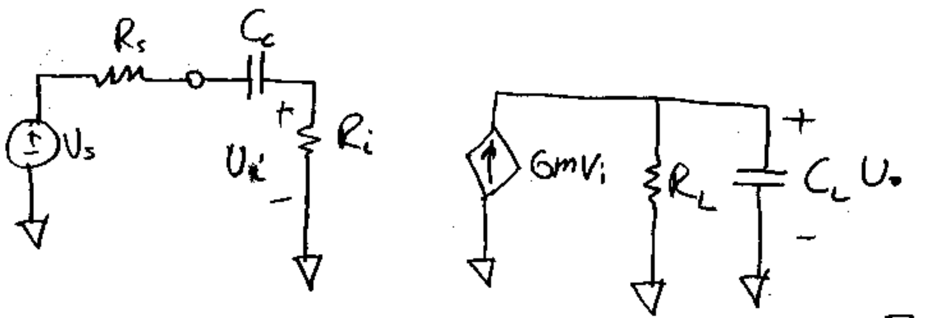
$1 + \frac{s}{10^4} \leftarrow \omega_{p_2}$

Zero is at 100k

$1 + \frac{s}{10^5} \leftarrow \omega_{z_1}$

Phase

7.14)



$$Z_L = R_L // \frac{1}{sC}$$

a) $V_{out} = G_m V_i Z_L \qquad \frac{V_{out}}{V_i} = G_m Z_L$

$$V_i = V_s \frac{R_i}{R_s + \frac{1}{sC_c} + R_i} = V_s \frac{sR_iC_c}{1 + s(R_i + R_s)C_c}$$

$$\frac{V_i}{V_s} = \frac{sR_iC_c}{1 + s(R_i + R_s)C_c}$$

$$\frac{V_o}{V_s} = G_m Z_L \left(\frac{sR_iC_c}{1 + s(R_i + R_s)C_c} \right)$$

$$Z_L = \frac{1}{\frac{1}{R_L} + sC_L} = \frac{R_L}{1 + sR_LC_L}$$

$$\frac{V_o}{V_s} = G_m \left(\frac{R_L}{1 + sR_LC_L} \right) \left(\frac{sR_iC_c}{1 + s(R_i + R_s)C_c} \right)$$

To find A_m short all caps.

b) $$A_m = \left(\frac{R_i}{R_i + R_s} \right) \left(G_m R_L \right)$$

$$F_L = \frac{s}{s + \frac{1}{C_c(R_s + R_i)}} \qquad F_H = \frac{1}{1 + sR_LC_L}$$

$A_m = 20\,dB$ So $20\log(A_m) = 20\,dB$

$A_m = 10\ \frac{V}{V}$

Since

$$A_m = \left(\frac{R_i}{R_i + R_s}\right)(G_m R_L)$$

$$10 = \left(\frac{100k}{100k + 20k}\right)(G_m(10k))$$

$$G_m = 1.2\ \frac{mA}{V}$$

d) $f_L \leq 10Hz$

$$f_L = \frac{1}{2\pi C_c (R_s + R_i)} \quad \text{So} \quad C_c = \frac{1}{2\pi f_L (R_s + R_i)}$$

$$C_c = \frac{1}{2\pi(10)(20k + 100k)} = 0.13\mu F$$

$$C_{c\,min} = 0.13\mu F$$

e) $f_H > 1mHz$

$$f_H = \frac{1}{2\pi R_L C_L} \Rightarrow C_L = \frac{1}{2\pi R_L f_H}$$

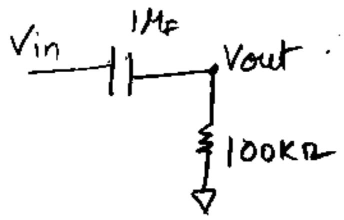
$$C_L = \frac{1}{2\pi(10k)(1mHz)} = 15.9pf \approx 16pf$$

$$C_L = 16pf$$

7.15.

$A_M = 100\, V/V$ (Given)

$F_L(s)$ is determined from the input circuit



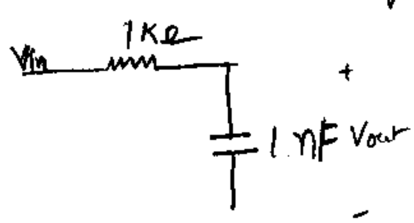
By voltage division.

$$V_{out} = V_{in}\left(\frac{R}{R+\frac{1}{sC}}\right)$$

$$\frac{V_{out}}{V_{in}} = \frac{10^5}{10^5 + \frac{1}{s\times10^{-6}}}$$

$$= \frac{s}{s+10}.$$

$F_H(s)$ is determined from the output circuit



$$V_{out} = V_{in}\left(\frac{\frac{1}{sc}}{R + 1/sc}\right)$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{s10^{-9}}}{R + \frac{1}{s10^{-9}}}$$

$$= \frac{1}{1 + \frac{s}{10^{6}}}$$

$$\omega_L(s) = 10 \text{ rad/s}$$

$$\omega_H(s) = 10^{6} \text{ rad/s}.$$

7.19.

<u>Low Frequency Response</u>.

By dominant pole approximation.

$$f_L \simeq 90\,Hz$$

By square root sum of squares.

$$f_L = \sqrt{10^2 + 20^2 + 90^2 - 2\times15^2}$$

$$= 90.3\,Hz.$$

High Frequency response:

By dominant Pole approximation $f_H \approx 15\ KHz$.

By square root sum of squares

$$f_H = \sqrt{\left(\frac{1}{15}\right)^2 + \left(\frac{1}{40}\right)^2}$$

$$= 14\ KHz.$$

7.26) $R = 100k$

$R_{in} = 1.2MEG$

$g_m = 2 mA/V$

$R_L' = 12K$

$C_{gs} = C_{gd} = 1pF$

$A_m = -22.2 \frac{V}{V}$

$f_{-3dB} =$

$$A_m = \frac{V_o}{V_{in}} = \frac{-R_{in}}{R_{in} + R} \left(g_m R_L' \right)$$

$$= \left(\frac{-1.2MEG}{1.2MEG + 100k} \right) \left(2mA/V \, (12K) \right)$$

$$= -22.2 \frac{V}{V}$$

$$R_{gs} = R_{in} // R = 1.2MEG // 100K = 92.3K$$

$$\tau_{gs} = C_{gs} R_{gs} = (1pF)(92.3K) = 92.3nS$$

$$R_{gd} = R' + R_L' + g_m R_L R' \qquad R' = R_{in} // R = 92.3K$$

$$= 92.3k + 12k + 2mA \cdot 92.3k = 2.32MEG$$

$$\tau_{gd} = C_{gd} R_{gd} = 1pf \, (2.32MEG) = 2320ns$$

$$\omega_H = \frac{1}{\tau_{gs} + \tau_{gd}} = \frac{1}{92.3nS + 2320nS} = 414.5k \text{ rad/sec}$$

$$f_H = \frac{\omega}{2\pi} = \frac{414.5k \text{ rad/sec}}{2\pi} = 66kHz$$