

Conduction: $q_x = -kA \frac{dT}{dx}$

Convection: $q = hA(T_s - T_\infty)$

Radiation: $q_r = \epsilon A \sigma T^4$

Diffusion equations:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad \text{uniform properties, } \alpha = \frac{k}{\rho C_p}$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left(\frac{\partial T}{\partial \phi} \right) + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad \text{Cylindrical coordinates}$$

Ohms Law Analogy, $q = \frac{\Delta T}{R_t}$

Plane wall: $R_{t,cond} = \frac{L}{kA}$, $R_{t,conv} = \frac{1}{hA}$

Cylinder: $R_{t,cond} = \frac{\ln(r_2/r_1)}{2\pi Lk}$, $R_{t,conv} = \frac{1}{2\pi r L h}$

Fins of uniform cross section: $\theta = T - T_\infty$ $m^2 = hP/kA_c$ $M = (hPkA_c)^{1/2} \theta_b$

Infinite fin: $\theta = \theta_b e^{-mx}$ $q_f = M$

Adiabatic tip: $\theta = \theta_b \frac{\cosh[m(L-x)]}{\cosh(mL)}$ $q_f = M \tanh(mL)$

Prescribed tip: $\theta = \theta_b \frac{(\theta_L/\theta_b) \sinh(mx) + \cosh[m(L-x)]}{\sinh(mL)}$ $q_f = M \frac{\cosh(mL) - (\theta_L/\theta_b)}{\sinh(mL)}$

Active tip: $\theta = \theta_b \frac{(h/mk) \sinh[m(L-x)] + \cosh[m(L-x)]}{(h/mk) \sinh(mL) + \cosh(mL)}$ $q_f = M \frac{\sinh(mL) + (h/mk) \cosh(mL)}{\cosh(mL) + (h/mk) \sinh(mL)}$

All fins: Effectiveness, $\mathcal{E}_f = \frac{q_f}{hA_b \theta_b}$ Efficiency, $\eta_f = \frac{q_f}{hA_f \theta_b}$

Lumped Capacitance: $Bi = hL_c/k < 0.1$ $Fo = \alpha t/(L_c)^2$ $\frac{T - T_\infty}{T_i - T_\infty} = e^{-t/\tau_t}$ $\tau_t = \frac{\rho V C_p}{hA_s}$ $Q = \rho V C_p \theta_i (1 - e^{-t/\tau_t})$

Semi-infinite solid:

imposed temperature: $\frac{T - T_s}{T_i - T_s} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$

Finite difference methods:

Explicit, 1-D unsteady: $T_m^{p+1} = Fo(T_{m+1}^p + T_{m-1}^p) + (1 - 2Fo) T_m^p$ $Fo = \alpha \Delta t / (\Delta x)^2 \leq 1/2$ for stability

Convection, Steady flow over a flat plate: $Pr = C_p \mu / k = \mu / (\rho \alpha)$ $Re_x = \rho u_\infty x / \mu$ $Nu_x = hx / k_f$

Laminar: $\delta = 5x Re_x^{-1/2}$ $C_{f,x} = 0.664 Re_x^{-1/2}$ $\overline{C_{f,L}} = 1.328 Re_L^{-1/2}$ $Nu_x = 0.332 Re_x^{1/2} Pr^{1/3}$ $\overline{Nu_L} = 0.664 Re_L^{1/2} Pr^{1/3}$

Mixed laminar and turbulent for $Re_{x,c} < Re_x < 10^8$: $\delta = 0.37x Re_x^{-1/5}$ $C_{f,x} = 0.0592 Re_x^{-1/5}$ $Nu_x = 0.0296 Re_x^{4/5} Pr^{1/3}$

Mixed laminar and turbulent with $Re_{x,c} = 5 \times 10^5$: $\overline{C_{f,L}} = 0.074 Re_L^{-1/5} - 1742 / Re_L$ $\overline{Nu_L} = (0.037 Re_L^{4/5} - 871) Pr^{1/3}$

Note: If the laminar region is small, the constants 1742 and 871 in the above equations become zero.

